Numeric Response Questions

Continuity and Differentiability

Q.1 If $f(\text{x})$ be a continuous function defined for $1 \leq \text{x} \leq 3, f(\text{x}) \in \text{Q} \forall \text{x} \in [1,3], f(2) = 10$, then find value of $f(1.8)$ (where Q is a set of all rational numbers).

Q.2 If the function $f(x) = \begin{cases} \frac{\sin \sqrt[3]{x} \log(1+3x)}{(\tan^{-1} \sqrt{x})^2 (e^{5\sqrt[3]{x}}-1)}, & x \neq 0 \\ a, & x = 0 \end{cases}$, is continuous at $x = 0$ then find a.

Q.4 Find the value of p, for which $f(x) = \left\{\sin\left(\frac{x}{p}\right) \log_e \left\{1 + \left(\frac{x^2}{3}\right)\right\}\right\}$, $x \neq 0$ is continuous at $x = 0$,

Q.5 If $f(x) = \begin{cases} \frac{1-\sqrt{2}\sin x}{\pi-4x}; x \neq \frac{\pi}{4} \\ a \quad ; x = \frac{\pi}{4} \end{cases}$ is continuous at $\text{x} = \frac{\pi}{4}$ then find value of a.

Q.6 If $f(x) = \begin{cases} \frac{2^x-1}{\sqrt{1+x}-1}, & -1 \leq x < \infty, x \neq 0 \\ k, & x = 0 \end{cases}$ is continueus everywhere, then k is equal to loge λ then find λ

Q.7 Find the value of $f(0)$, so that the function $f(x) = \frac{2x-\sin^{-1} x}{2x+\tan^{-1} x}$ is continuous at each point in its domain.

Q.8 Let $f(\text{x}) = (\sin \text{x})^{\frac{1}{\pi-2x}}, \text{x} \neq \frac{\pi}{2}$, If $f(\text{x})$ is continuous at $\text{x} = \frac{\pi}{2}$ then find the value of $f\left(\frac{\pi}{2}\right)$.

Q.9 Find the number of points at which function $f(x) = |x - 0.5| + |x - 1| + \tan x$, does not have a derivative in the interval $(0,2)$

Q.10 Find the value of b such that the function $\text{f}(x) = \begin{cases} ax + 3, & x \geq 1 \\ x^2 + b, & x < 1 \end{cases}$ is continuous and differentiable at $x = 1$

Q.11 If $f(x) = \begin{cases} e^x & x < 1 \\ a - bx & x \geq 1 \end{cases}$ is differentiable for $x \in \text{R}$ then find value of $\left(\text{a} - \frac{\text{b}}{\text{e}}\right)$.

Q.12 Find number of non-differentiable point for $f(x) = \min(\sin x, \cos x)$ is (if $x \in (0,4\pi)$).

Q.13 Find the number of points where $f(x) = ||x| - 1|$ is not differentiable.

Q.14 If the function $g(x) = \begin{cases} k\sqrt{x+1}, & 0 \leq x \leq 3 \\ mx + 2, & 3 < x \leq 5 \end{cases}$ is differentiable, then find the value of $k + m$.

Q.15 Suppose $\Omega(x)$ is differentiable at $x = 1$ and $\lim_{h \to 0} \frac{1}{h} \rho(1 + h) = 5$, then find $f'(1)$.

ANSWER KEY

1. 10.00	**2.** 0.60	**3.** 6.00	**4.** 4.00	**5.** 0.25	**6.** 4.00	**7.** 0.33
8. 1.00	**9.** 3.00	**10.** 4.00	**11.** 1.00	**12.** 4.00	**13.** 3.00	**14.** 2.00
15. 5.00						

Hints & Solutions

1. $\therefore$ f(x) $\in$ Q = f(x) is constant function

$\therefore$ f(2) = 10

$\Rightarrow$ f(1.8) = 10

2. $$\lim_{x\to0}\left(\frac{\sin x^{1/3}}{x^{1/3}}\cdot\frac{\log(1+3x)}{3x}\cdot\left(\frac{\sqrt{x}}{\tan^{-1}\sqrt{x}}\right)^2\cdot\frac{5\sqrt[3]{x}}{e^{5\sqrt[3]{x}}-1}\cdot\frac{3}{5}\right)$$

$= 1.1.(1)^2.1.\frac{3}{5} = \frac{3}{5}$

$\therefore$ then function f is continuous at x = 0 if

$a = \frac{3}{5}$

3. $$\lim_{x\to0^-} f(x) = \lim_{x\to0^-}\frac{3}{x^2}\sin 2x^2$$

$$= 6\lim_{x\to0}\frac{\sin 2x^2}{2x^2} = 6$$

$$\lim_{x\to0^+} f(x) = \lim_{x\to0^+}\frac{x^2+2x+c}{1-3x^2} = \frac{c}{1} = c$$

Hence for f to be continuous c = 6.

4. $$\lim_{x\to0}\frac{(4^x-1)^3}{\sin(x/p)\log_e\left(1+\frac{x^2}{3}\right)} = 12\ (\log_e 4)^3$$

$$\lim_{x\to0}\frac{\left(\frac{4^x-1}{x}\right)^3 x^3}{\left(\frac{\sin(x/p)}{(x/p)}\right)\left(\frac{x}{p}\right)\left(\frac{\log(1+(x^2/3))}{x^2/3}\right)x^2/3}$$

$= 12(\log_e 4)^3$

$3p(\log_e 4)^3 = 12(\log_e 4)^3$

p = 4

5. (L' Hospital Rule)

For continuous value = limit

$$\Rightarrow a = \lim_{x\to\pi/4}\frac{1-\sqrt{2}\sin x}{\pi-4x}$$

$$\Rightarrow a = \frac{1}{4}.$$

6. $$k = \lim_{x\to0}\frac{2^x-1}{\sqrt{1+x}-1}$$

$$= \lim_{x\to0}\frac{2^x-1}{x}\times(\sqrt{1+x}+1)$$

= 2 log 2 = log 4

7. $$f(0) = \lim_{x\to0}\frac{2x-\sin^{-1}x}{2x+\tan^{-1}x},\ \frac{0}{0}$$

$$= \lim_{x\to0}\frac{2-\frac{1}{\sqrt{1-x^2}}}{2+\frac{1}{1+x^2}} = \frac{1}{3}$$

8. If function is continuous at $x = \pi/2$.

$$f\left(\frac{\pi}{2}\right) = \lim_{x\to\frac{\pi}{2}} f(x)$$

$$= \lim_{x\to\frac{\pi}{2}}(\sin x)^{\frac{1}{\pi-2x}},\ 1^\infty$$

$$= \lim_{x\to\frac{\pi}{2}} e^{\frac{1}{\pi-2x}(\sin x-1)};\ \left(\frac{0}{0}\right)$$

$$= \lim_{x\to\frac{\pi}{2}} e^{\frac{\cos x-0}{0-2}};\ \text{(by using L' Hospital rule)}$$

$$= e^{\frac{0}{-2}} = 1$$

9. We have, $y = |x - a|$ is not differentiable at $x = a$

$\therefore f(x)$ is not differentiable at $x = \frac{1}{2}$ and $x = 1$

Also $\tan x$ is not differentiable at $x = \frac{\pi}{2}$

Hence $f(x)$ is not differentiable at three points in (0, 2)

10. Since the function is continuous at $x = 1$, we have

$\lim_{h\to 0} f(1+h) = \lim_{h\to 0} f(1-h) = f(1)$

Now,

$\lim_{h\to 0} f(1-h) = \lim_{h\to 0} [(1-h)^2 + b] = 1 + b$

$\lim_{h\to 0} f(1+h) = \lim_{h\to 0} [a(1+h)+3]$

$= a + 3$ and $f(1) = a + 3$

Hence, $a + 3 = 1 + b$, or $b = a + 2$

Since the function is differentiable at $x = 1$, we have $f'(1^+) = f'(1^-)$.

We obtain

$$f'(1^+) = \lim_{h\to 0} \frac{1}{h}[f(1+h) - f(1)]$$

$$= \lim_{h\to 0} \frac{1}{h}[\{a(1+h)+3\} - (a+3)]$$

$$= a$$

$$f'(1^-) = \lim_{h\to 0} \frac{1}{(-h)}[f(1-h) - f(1)]$$

$$= \lim_{h\to 0} \frac{1}{(-h)}[\{(1-h)^2 + b\} - (a+3)]$$

$$= \lim_{h\to 0} \frac{1}{(-h)}[\{1 - 2h + h^2 + b\} - (1+b)]$$

$$= \lim_{h\to 0} \frac{1}{(-h)}[h^2 - 2h] = 2$$

Therefore, $a = 2$ and $b = a + 2 = 4$

11. As f(x) is differentiable at x = 1

so it will be continuous also

so $a - b = e$ ... (1)

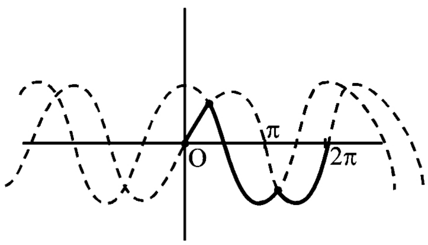
as f(x) is differentiable at x = 1

so $-b = e$ ... (2)

from (1) and (2)

$b = -e, a = 0$

12.



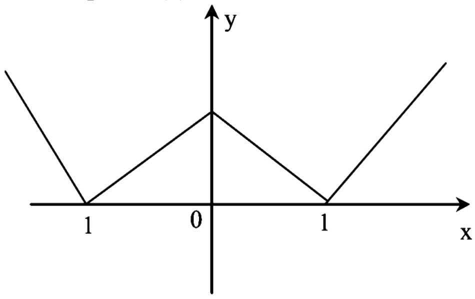
point of discontinuity in $(0, 2\pi) = 2$

so point of discontinuity in $(0, 2n\pi) = 2n$

(continuous)

13. $f(x) = ||x| - 1|$

Graph of f(x)



so f(x) is not differentiable at $x = -1, 0, 1$

14. Since function is differentiable at x = 3

it must be continuous at x = 3, also

$f(3) = f(3 + h)$

$= f(3 - h)$

$k\sqrt{3+1} = \lim_{h\to 0} m(3+h) + 2$

$= \lim_{h\to 0} k\sqrt{3-h+1}$

$2k = 3m + 2 = 2k$ ... (1)

Now differentiable at x = 3

$f'(3 + h) = m$

$f'(3-h) = \frac{d}{dx}(k\sqrt{x+1})$

$= k\frac{1}{2\sqrt{x+1}}$

at x = 3, $\frac{k}{4}$

hence $m = \frac{k}{4}$

$k = 4m$ ... (2)

Solving (1) and (2) we get

$m = \frac{2}{5}, k = \frac{8}{5}$

$k + m = \frac{2}{5} + \frac{8}{5} = 2$

15. $f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$

$\because$ f(x) is differentiable at x = 1

$\therefore$ it is continuous at x = 1 also

$\therefore f(1) = \lim_{x\to 1} f(x) = \lim_{h\to 0} f(1+h)$

$f(1) = \lim_{h\to 0} \frac{h\, f(1+h)}{h} = 0 \times 5 = 0$

$= \lim_{h\to 0} \frac{f(1+h) - 0}{h} = 5$